Operators in Quantum Mechanics

The Concept of Operators is very important in wave mechanics.

Algebra of Operators:

If $\hat{A}$ and $\hat{B}$ are two operators and f is the function upon which they operate, then

$$(\hat{A} + \hat{B}) f = \hat{A}f + \hat{B}f$$

and $$(\hat{A} - \hat{B}) f = \hat{A}f - \hat{B}f$$

Thus, we can generate new operators $\hat{A}+\hat{B}$ and $\hat{A}-\hat{B}$ by adding and substracting $\hat{A}$ and $\hat{B}$. Also,

$$\hat{A} + \hat{B} = \hat{B} + \hat{A}$$

$$\hat{A} - \hat{B} = -\hat{B} + \hat{A}$$

The multiplication of operators can be carried out by doing successive operations with two or more operators on a function.

If $\hat{A}$ and $\hat{B}$ are two operators and f is their Operand, then we can obtained the quantity $\hat{A}\hat{B}f$ as follows: —

We first operate on f with $\hat{B}$ to obtain f', that is, $\hat{B}f = f'$,

Then f' is operated upon $\hat{A}$ to obtain f'' that is, $\hat{A}f' = f''$

Thus, we have

$$\hat{A}\hat{B}f = f''$$

It should be remembered that the order of application of operators is always from right to left as they are written.

Application of the same operator several times in succession is written with a power.

Thus we have,

$$\hat{A}\hat{A}f = \hat{A}^2 f$$

Commutation of Operators: —

The Algebra of Operators should be distinguished from the algebra of Ordinary numbers. If a and b are two numbers, then we know that

$$a \times b = b \times a$$

However, if $\hat{A}$ and $\hat{B}$ are two operators, then their Product $\hat{A}\hat{B}$ may or may not be equal to $\hat{B}\hat{A}$

If $\hat{A}\hat{B} = \hat{B}\hat{A}$ then the Commutator of the two operators is defined as,

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = 0$$

It will be clear by example $[\hat{A}\hat{B}] = -[\hat{B}, \hat{A}]$

i.e.,

$$[\hat{A}, \hat{B}] = [\hat{A}\hat{B} - \hat{B}\hat{A}] = -(\hat{B}\hat{A} - \hat{A}\hat{B}) = -[\hat{B}, \hat{A}]$$

Expression for Operators: — Expression for operators can be obtained by operating with the operators on an operand and removing the operand at the end of operation.